Understanding the Three-Dimensional Conceptual Space of Fitts' Aimed-movement Paradigm

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November 2000

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Abstract

Fitts' law is a well known empirically-based relation which predicts aimed-movement time (*MT*) from target distance (*D*) and target width (*W*). Fitts' demonstration that *MT*, within limits, depends essentially on the ratio *D*/*W* implies a scale invariance that reduces the paradigm from three dimensions (*MT*, *D*, and *W*) to two (*MT* and *D*/*W*). This reduction, however, is legitimate only for narrow ranges of scale variations, a limitation that appears to have been overlooked so far. This paper advocates an explicit three-dimensional construal of Fitts' paradigm involving not only the speed (*MT*) and the relative amplitude (*D*/*W*), but also the absolute amplitude (*D*), or scale of movements. Not only is this three-dimensional description of Fitts' paradigm a technical necessity for the classic study of Fitts' law, but it paves the way for a more complete modeling of aimed-movement performance and suggests a promising adaptation of Fitts' paradigm to the recently emerged problem of target selection in zooming interfaces.

Contents

1. Introduction

This article is about the amplitude of human hand movements, a subject that is treated within the conceptual framework elaborated by Paul Fitts (1954), the discoverer of Fitts' law. Our movements, which Bernstein (1967) viewed as morphological objects, can be characterized by their shape as well as their size. Whether the goal is to generate a continuous trajectory (as in drawing) or, more simply, to reach a discrete target with the tip of some hand-held pointer (as in Fitts' experiments), any movement task can, to a large extent, be scaled up or down without altering its essential morphological characteristics.

To re-scale an aimed-movement task simply amounts to changing target distance and target width proportionally. But it is only from the experimenter's viewpoint that such a change is simple. The reproduction of the same movement at different scales involves dramatic qualitative changes in one's muscular and skeletal machinery, of which we are normally unaware—for example, scaling up the movement task may require the shoulder and the elbow joints to replace the fingers and the wrist (Lacquaniti, Ferrigno, Pedotti, Soechting, & Terzuolo, 1987). However, what happens behind the stage, in the high-dimensionality angular space of the effectors is a question, however important for human movement science (e.g., Morasso & Tagliasco, 1986), that is beyond the scope of Fitts' aimed-movement paradigm, which cannot do more than provide a convenient, albeit reductive, experimental context. The conceptual framework elaborated by Fitts considers movement exclusively in *work space*—the low-dimensionality linear space covered by the arm's endpoint, and also the functionally crucial space in which organism-environment interactions take place (Mottet, Guiard, Bootsma, & Ferrand, in press; Saltzman & Kelso, 1987).

Fitts' aimed-movement paradigm makes things really simple. Not only does it reduce the movement task to that of reaching a single environmental location with a single body point, but it deliberately ignores all the complexity of the underlying biomechanics, to consider exclusively the motion of a single point in work space, be it a finger tip, a stylus tip, or a screen cursor. This radical simplification strategy, reminiscent of particle-motion modeling in classical mechanics, has proved quite successful. The paradigm, notably, has made it possible to establish Fitts' law, admittedly one of the most general and robust regularities in the whole field of experimental psychology (see Section 2.3). However, rather surprisingly, we will see that the notion of movement amplitude still suffers some degree of obscurity in Fitts' paradigm. It will be shown that the ambiguity of the definition of amplitude that has been used so far has caused conceptual muddles and troublesome experimental errors.

The paper will proceed as follows. After a glossary of the main variables that need to be distinguished in Fitts' paradigm (see Table 1), Section 2 will present a simple, mathematically inspired analysis aimed at showing that the paradigm, contrary to its current understanding, is irreducibly three-dimensional, involving the variables of movement speed, absolute amplitude (or scale), and relative amplitude (distance scaled to error tolerance).

Sections 3 and 4 will focus on the statuses and the actual influences of absolute and relative amplitude, the paradigm's two independent variables. From a review of the literature, it will be suggested that these two variables exert comparable impacts on aimed-movement performance, leading to the view that absolute amplitude needs to be taken into consideration just like relative amplitude. Also, the utility of rephrasing relative amplitude in terms of the subjective notion of difficulty will be questioned, on the grounds that both absolute and relative amplitude bear a close relationship with this hypothetical intermediate variable, and that in either case the relationship is non-linear.

Section 5 will present a critical analysis of the experimental design that has been used uninterruptedly since Fitts (1954). It will be shown that this design confounds the effects of absolute and relative amplitude, thereby exposing the assessment of Fitts' law to the risk of being contaminated, to an uncontrolled extent, by an unwanted influence of the scale factor.

Section 6 will present the main implications that can be drawn from the proposed three-dimensional understanding of Fitts' paradigm. First, one's method of assessing Fitts' law can be improved, second, one's approach to the modeling of aimed-performance can be enriched from two to three dimensions, and third, Fitts' aimed-movement paradigm can be generalized to the recently emerged case of pointing in multi-scale electronic worlds.

Table 1. A Glossary of the Main Variables Involved in Fitts' Aimed-movement Paradigm

Variable Name	Definition	Physical Dimension
Independent variables (environmental characteristics)		
Target Distance (D)	The distance separating target center from starting point	Length
Target Width (W)	Target size, specifying the prescribed tolerance interval	Length
Relative Target Dist. (D/W)	Target distance scaled to target width	Dimensionless
Index of Difficulty (ID)	Some mathematical function of D/W, e.g., $\log_2 (2D/W)$	Dimensionless
Dependent measures (movement characteristics)		
Movement Time (MT)	The duration of the aimed movement	Time
Movement Amplitude (A)	The distance actually covered by the movement	Length
Effective Target Width (W_e)	The dispersion, over repetitions, of movement endpoints (*)	Length

Relative Amplitude (A/W_e)	Movement amplitude scaled to movement endpoint dispersion	Dimensionless
Index of Effective Diffic. (ID_e)	Some mathematical function of A/W_e, e.g., $\log_2 (2A/W_e)$	Dimensionless

(*) Note. More specifically, W_e is the calculated W such that, given the observed dispersion of movement endpoints, a certain pre-specified error rate would have occurred.

2. The Three-Dimensional Conceptual Space of Fitts' Paradigm

This section examines the issues of the number and identity of the variables involved in Fitts' aimed-movement paradigm. An experimental paradigm can be viewed as the mathematical space in which abstract relationships can be conceived. The paradigm's dependent and independent variables are the dimensions of this space.

Fitts' aimed-movement paradigm could be said to be four-dimensional in view of its basic ingredients: a pair of dependent measures, speed (MT) and accuracy (percentage of target misses), and a pair of manipulated variables, target distance (D) and target width (W). However, we will see that Fitts' law research has led to the reduction of each of these two pairs of variables to a single variable. So the currently received version of the paradigm involves an essentially two-dimensional conceptual space in which MT is represented as a function of movement difficulty, based on the ratio D/W.

It will be argued, however, that Fitts' paradigm cannot work satisfactorily with fewer than *three* dimensions. While speed and accuracy measurements can be legitimately reduced to a single dependent variable, we will see that no such simplification can be achieved on independent variables. Concerning the identity of the dimensions, it will be shown that the irreducible two degrees of freedom (df) that experimenters have at their disposal cannot be formulated in terms of D and W, owing to inescapable confounds. The real independent variables of the paradigm will be shown to be the relative and the absolute amplitude of movement.

2.1. Reducing Speed and Accuracy to a Single Dependent Variable

A speeded aimed-movement task inevitably gives rise to occasional target misses. It has been repeatedly observed that, as the task is made more difficult, the probability of errors generally increases (e.g., Fitts, 1954; Crossman & Goodeve, 1963/83, Welford, 1968). Conversely, when the task becomes very easy, it is a common observation that people fail to exploit the whole error tolerance made available to them, with the spatial dispersion of their movement endpoints spread typically over an interval smaller than W (Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1988). Taken together, these two effects can be described as a range effect. That is, movement precision—as can be assessed from the ratio of the mean and the standard deviation of amplitude—typically varies over a smaller range than recommended by experimenters via their manipulation of task difficulty (Guiard & Ferrand, 1998).

As noted by Welford (1968), such effects are liable to bias the assessment of Fitts' law, and therefore *MT* needs to be corrected for errors. A further concern is that performance is hard to evaluate if it is defined in terms of both speed and accuracy, because it can be affected differently on these two dimensions by experimental manipulations. The correction of *MT* for errors eliminates this problem in advance by neutralizing the variations of accuracy, which then can be safely ignored.

Specifically, the solution introduced by Welford (1968) consists of replacing nominal tolerance W with effective tolerance W_e estimated from the standard deviation of movement endpoints[1] and computing the *ID* on the basis of D/W_e. This amounts to calculating, for each level of the *ID*, the *MT* that would have been obtained, had the participant stuck to some constant, low level of error rate. In fact, for Welford's procedure to be complete, one must also check if, on average, movement amplitude (A) equaled the prescribed distance D in each condition—an equality whose probability decreases as the task becomes easier because, to ensure a hit, the movement no longer needs to cover the whole prescribed amplitude (Guiard & Ferrand, 1998). In sum, the simple solution is to replace the nominal *ID*, computed from D/W, with an index of *effective* difficulty (ID_e), computed from the more realistic ratio A/W_e. Such a procedure makes it possible to ignore accuracy variations and thus one is left with a single dependent variable, *MT*.

2.2. The Question: Counting and Identifying the Paradigm's Independent Variables

While most of our movements take place in 3D space, the experimental paradigm introduced by Fitts (1954), in line with an experimental stream which can be traced back to Woodworth (1899), reduces the problem of aimed movement to a single spatial dimension. Fitts' parsimonious conceptualization only considers three points on a continuum[2] (see Figure 1): one point to specify the current, or starting position and another two to specify a target interval allowing some tolerance for error. It is noteworthy that the smallest possible number

[1] An alternative, less reliable, procedure is to compute W_e from the observed frequencies of undershoots, hits, and overshoots.

[2] As observed by Fitts (1954, Footnote 4, p. 387), this continuum need not be spatial length. The same rationale applies, beside amplitude, to any dimension of movement like its direction or its force—or even, as astutely remarked by Woodworth (1899), to the vocal pitch of a singer.

of points needed to capture the problem of aimed movement in 1D space is indeed three, and so the paradigm cannot be reduced to any simpler form.

These three points define two relevant lengths that can be manipulated experimentally, target distance (*D*) and target width (*W*). Length *D*, usually measured from starting position to the center of the target interval, serves to constrain movement amplitude (*A*), while length *W* serves to constrain the spatial variability of movement endpoints over repetitions.

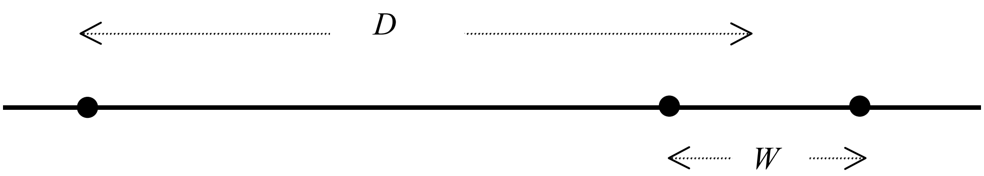

Figure 1. The two basic lengths involved in Fitts' aimed-movement paradigm.

The question addressed here is, What variables can a student of human aimed movement manipulate independently in the extremely simple situation depicted in Figure 1 or, equivalently, What are the dimensions of the conceptual space involved in Fitts' paradigm? It will become apparent below that, if Fitts' law is taken into account, this question is far less trivial than it may seem at first. Our analysis must start with a brief reminder of Fitts' law.

2.3. Fitts' Law in a Generic Form

Fitts demonstrated that in target acquisition tasks movement time (*MT*) is essentially dependent on the ratio D/W. Fitts (1954; Fitts & Peterson, 1964) formulated the law as

$$MT = a + b \log_2(2D/W), \qquad (1)$$

with *a* and *b* standing for adjustable constants and $\log_2(2D/W)$ representing the task's index of difficulty (*ID*).[3]

Since Fitts' pioneering work, a number of alternative formulations of the law have been proposed (Accot & Zhai, 1997; MacKenzie, 1992; Meyer, Smith, Kornblum, Abrams, & Wright, 1990; Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979; Welford, 1968). Notwithstanding the utility of the proposed amendments, it must be noted that theorists have introduced only minor changes to Fitts' model, whose two primary assumptions have been retained. First, all authors have agreed to hold the ratio D/W as the only determiner of task

[3] For convenience, this version of the *ID* will be used by default throughout this article.

difficulty. Second, all authors have assumed a linear relationship between *MT* and the proposed *ID*. That is, it has been unanimously admitted so far that

$$ID = \mathrm{f}\,(D/W), \tag{2}$$

with f standing for some simple—linear, logarithmic, or power—mathematical function, and that

$$MT = a + b\, ID. \tag{3}$$

Taken together, Equation 2, which defines task difficulty without specifying any particular function, and Equation 3, which states a linear dependency of *MT* upon the *ID*, can be taken as the generic formulation of Fitts' law. We will repeatedly refer to this formulation in the rest of this paper.[4]

2.4. Amplitude and Tolerance: Two Non-Independent Variables

Equations 2 and 3 have an important implication that seems to have attracted little attention so far. This implication is that *D* and *W cannot* work as independent variables in a Fitts' law experiment. What should be realized is that if *D* is manipulated at a constant level of *W*, then by Equation 2 the *ID* will vary too and hence there will be no way, in the analysis of *MT*, to disentangle the effect of the *ID* from that of *D*. For example, for a constant $W = 1$ cm, changing *D* from 20 cm to 40 cm yields a condition in which the movement is simultaneously more difficult and larger in amplitude, since both the numerator *D* and the ratio D/W change from 20 to 40. Likewise, if *W* is manipulated at a constant level of *D*, then the *ID* will again be affected, and so it will be unclear whether the effect observed on *MT* must be attributed to target width or difficulty.

The simple point being made here is that changing just the numerator or just the denominator of a ratio alters the ratio itself. From the moment this ratio is known to be influential (and this indeed is the core of the lesson learnt from Fitts' law research), it is unwise to ignore such a factor confound between *D* and the *ID*, or between *W* and the *ID*. The assessment of Fitts' law requires that variables *D* and *W* be manipulated orthogonally with each other, but neither of the two basic manipulations work since the ratio D/W, a very influential factor on its own, will inevitably vary at the same time. This leads us to the

[4] Schmidt et al. (1979) introduced a variant of Fitts' law of the form $W_e = \mathrm{f}(D, MT)$, with W_e standing for effective target width, defined as the spatial dispersion of movement endpoints around a target point—no tolerance being explicitly specified for error. A major characteristic of Schmidt et al.'s version of the paradigm is that movement tolerance and movement speed swap their roles, with *W* being measured as a dependent variable and *MT* being manipulated as an experimental variable. Schmidt et al.'s version of the paradigm will receive no specific treatment below because, from the standpoint adopted in the present paper, it does not seem to differ essentially from Fitts'.

following problem: What are the paradigm's variables that can be manipulated independently of any other variable, if this possibility exists at all.

2.5. The Paradigm's True Independent Variables: Absolute and Relative Amplitude

Below we will resort to a simple geometrical analogy to suggest that Fitts' paradigm has two irreducible independent variables, and that these are, not the amplitude and the tolerance as commonly assumed, but rather the relative and the absolute amplitude of the movement. Since movement difficulty is entirely captured by a ratio, it is analogous to shape in one-dimensional (1D) space.

Let us start with 2D geometry, the most familiar context for thinking of shape (Colton, 1998). It takes five df to uniquely specify a rectangle in 2D space, and shape can be defined as one of these. For example, as illustrated in the upper part of Figure 2, one may completely specify the rectangle ABCD in a Cartesian coordinate system by providing one number for figure shape (the aspect ratio AB/BC), one for figure size (the length of any segment, say AB), two for figure position (the x and y coordinates of any corner, say x_A and y_A), and a final one for figure orientation (the angle ϕ subtended by any segment relative to either axis, say the angle formed by the line AB and the Oy axis).

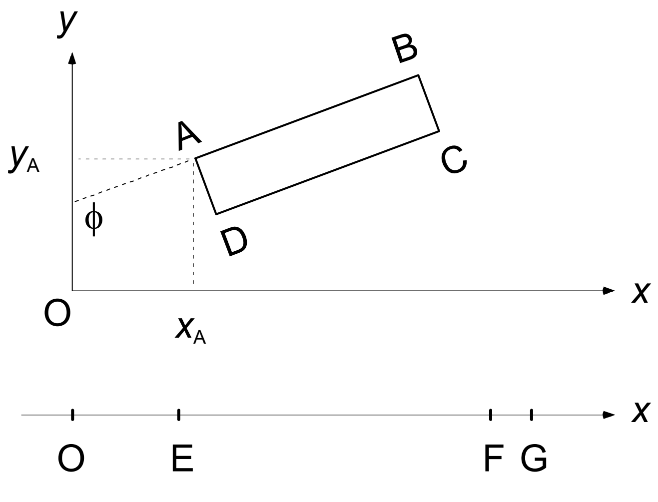

Figure 2. Specifying shape and scale in 2D space (above) and in 1D space (below). The set of three points E, F, and G below (with E representing the starting point, and F and G representing the target interval of a Fitts task) can be thought of as a 1D figure.

Defining an aimed-movement task in Fitts' paradigm is like defining a three-point figure in 1D space, as shown in the lower part of Figure 2. Three df are obviously involved but we can resort to different sets of three numbers to specify Figure EFG. One can simply express the figure's three abscissas x_E, x_F, and x_G. Another possibility, consistent with the traditional understanding of Fitts' paradigm, is to specify target distance, target width, and target location, that is, EF+½FG , FG, and OE, respectively. But there is a third possibility, which consists of specifying

(1) figure *shape*: the aspect ratio EF/FG or, equivalently, the ratio (EF+½FG)/FG to match the ratio D/W of Equation 2,

(2) figure *size*, or scale:[5] the length of any segment, say EF+½FG to match the usual definition of D, and

(3) figure *location*: the distance OE between the starting position and some origin. In practice, we will ignore this df, of little relevance to the problem at hand.

So, if we obviously need two df to specify a Fitts task (the absolute location of the target being ignored), it is noteworthy that these two df need not be conceptualized as D and W. The task can be thought of just as well in terms of task *shape*, specified by the ratio D/W, a dimensionless quantity, and task *scale*, specified by D, which has the physical dimension of length.[6] Even though the D and W description and the D/W and D description both specify a Fitts task completely, they are not equivalent for the aimed-movement paradigm, keeping Fitts' law in mind. If shape is an important characteristic of the figure, then we should prefer the latter description, because it identifies shape explicitly.

Table 2 shows the suggested correspondences. Movement difficulty rigorously corresponds to—being entirely determined by—*relative* movement amplitude, in the sense of amplitude scaled to tolerance, and this is an analogue to figure shape; task scale is equated with *absolute* movement amplitude—amplitude scaled to some external standard of length—and this is analogue to figure size.

[5] Note that the terms "size" and "scale" are treated in this paper as strict synonyms.

[6] The use of D (or A), rather than W (or W_e), as an index of movement scale will be justified in the next section.

Table 2. Operational definitions of the paradigm's two independent variables, along with their respective geometrical analogues

Variable name	*Operational definition*	*Geometrical analogue*
Movement difficulty	Relative amplitude D/W	Figure shape
Movement scale	Absolute amplitude D	Figure size

Importantly, movement difficulty and movement scale, unlike D and W, do qualify as *independent* variables for experimenting in the paradigm, keeping in mind the constraint of Fitts' law. As illustrated in Figure 3, varying task scale changes the absolute, but not the relative amplitude of the required movement: D (together with W) changes but the ratio D/W does not, thus keeping movement difficulty constant. Reciprocally, varying W at a constant level of D changes the relative, but not the absolute amplitude of the movement: the ratio D/W (and hence the *ID)* is made to change in the absence of any variation of D, thus keeping movement scale constant.

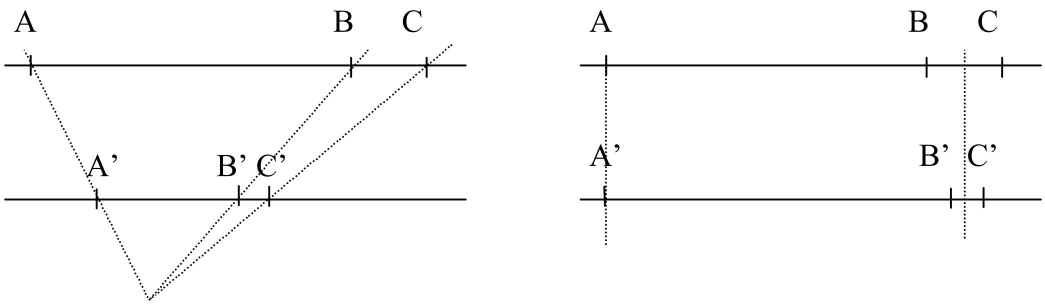

Figure 3. Manipulating movement scale and movement difficulty independently of each other in Fitts' aimed-movement paradigm. Point A marks the starting point, and the target is represented by the interval BC. *Left*: variation of movement scale at a constant level of difficulty. *Right*: variation of movement difficulty at a constant level of scale.

We must be clear about what it means to scale an aimed-movement task up or down. Consider two task conditions with the same D/W ratio, one with, say, D = 20 cm and W = 2 cm and the other with D = 40 cm and W = 4 cm—the latter task condition is a two-fold scaled up version of the former. To say that the two conditions involve the same *ID* is like saying that 20/2 = 40/4. But one should be aware of the dual meaning of the equal sign in the last statement. In fact, 20/2 and 40/4 are both equal and different. If one refers to the *rational number* involved (relative amplitude), then the equal sign denotes a mathematical equality. If, however, one considers the *fractions*, then the equal sign denotes an equivalence—namely, the shared capability of two different expressions to represent a certain rational number, whose simplest expression is 10/1 = 10. By recognizing that expressions like 10/1, 20/2, 40/4, 80/8, etc., obviously *are* different things (different fractions) which amount, in some specific sense, to the same thing (the same rational number), one recognizes the existence of the scale variable—that is, one recognizes that to specify 20/2 and 40/4 one need *two df*, as shown in Table 3.

Table 3. The Two Degrees of Freedom of Arithmetical Fractions.

		Ratio		
		1	10	100
	100	100/100	100/10	100/1
Scale (Numerator)	200	200/200	200/20	200/2
	300	300/300	300/30	300/3

Henceforth our new factorial description of Fitts' paradigm will be designated as the *Absolute vs. Relative Amplitude* (ARA) description,[7] as distinct from the currently accepted *Amplitude vs. Tolerance* (AT) description, which will be examined in greater detail in Section 5 below. It should be emphasized that distinguishing ARA and AT designs is not just a technical matter for the way in which we design our experiments closely reflects the way in which we conceptualize our research problems. At stake here is the identification of the essence of Fitts' aimed-movement problem.

3. Absolute Amplitude: The Influence of Movement Scale on Performance

In this section we discuss the definition of movement scale in the context of Fitts' paradigm and we ask about the actual influence of this factor on performance. It will be argued that scale can be conveniently quantified in Fitts' paradigm by the absolute amplitude of the movement, ignoring target width. Second, we will examine the fact that the independent variable of scale, in comparison with relative amplitude, has received little attention so far. The reason, we will propose, is because, according to Fitts' law, *MT* should obviously be scale independent (see Equations 2 and 3). But this cannot possibly be true, as we will see.

[7] This alternative factorial description of Fitts' paradigm that is proposed in the present paper might have been dubbed the "difficulty versus scale" description. However, as will be explained in Section 4, the equivalence of difficulty and relative amplitude does not seem warranted. Therefore, we will stick to the less fluid ARA label.

3.1. Defining Movement Scale as Absolute Movement Amplitude

Since any length measure can serve to quantify the scale of Figure EFG shown in Figure 2, one might think that *D* and *W* should do the job just as well. Note, however, that these two candidate indices of scale are equivalent only if shape is constant. How can the scale of two aimed-movement tasks be compared if their D/W ratios differ?

Suppose you want to compare, for movement scale, a given task condition to another with a larger *D* but a smaller *W*. Why should *W* be ignored to evaluate task size? An obvious solution would be to combine *D* and *W* in some way. It is common practice to characterize the size of rectangular objects like computer screens by taking the length of the diagonal $(AB^2 + BC^2)^{1/2}$. So we could quantify movement scale in Fitts' paradigm as $(D^2 + W^2)^{1/2}$.

However, it seems preferable to simply use target distance *D* to estimate the scale of the required movement. Recall that, outside of Fitts' law literature, it is a well established convention in human movement science to equate movement scale with movement amplitude defined as the largest spatial extension of the movement. For example, students of trajectory formation in handwriting usually take movement scale to be simply measured by the height of letters (e.g., Lacquaniti, Ferrigno, Pedotti, Soechting, & Terzuolo, 1987; Wright, 1993). Thus, recourse to $(D^2 + W^2)^{1/2}$ to quantify movement scale in the special context of Fitts' paradigm would have the drawback of breaking a useful correspondence with a widely shared notion of scale.

Note that the practical cost of ignoring tolerance for the estimation of scale in Fitts' paradigm seems quite moderate. The size of an elongated object is efficiently captured by its longest extent. For example, comparative anatomy takes the size of a bone to be its length, rather than a combination of its length and its thickness (Colton, 1998; Gould, 1977), and to characterize the size of a sky-scraper a simple height estimate works. Likewise, *D* is typically so much larger than *W*—the ratio D/W actually rises in an accelerated manner from 1 to about 500 as the *ID* varies from 1 to 10—that *W* can only contribute little to the square root index. Thus, the quantification of movement scale by movement amplitude or target distance can involve only a very small error, relative to the more comprehensive square root index. This error becomes quite negligible for *ID*s above 3 bits, as shown in Figure 4. Thus, in the rest of this paper, the scale of an aimed movement will be characterized simply by its amplitude (*A* or *D*).

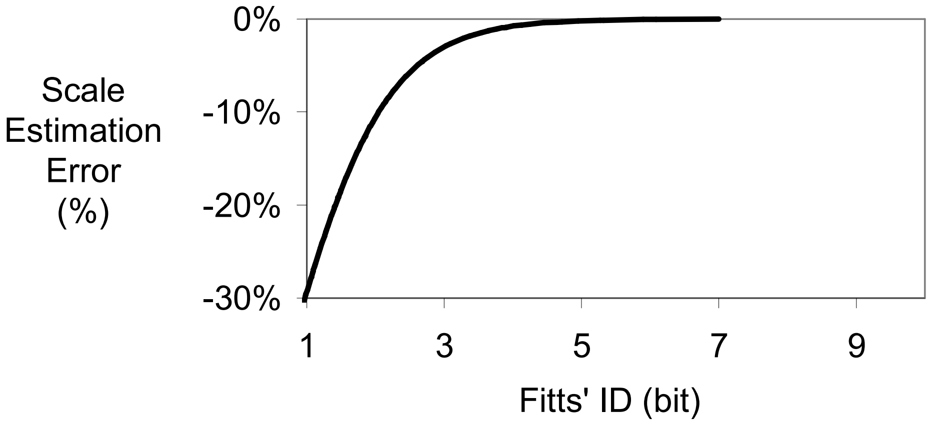

Figure 4. The difference between the estimate of scale as D and as $(D^2+W^2)^{1/2}$, in percentage of the latter, as a function of movement difficulty.

3.2. The Within-Limits Validity of Fitts' Law

Compelling logical arguments and abundant empirical evidence have forced researchers, since Fitts (1954), to make the reservation that Fitts' law is valid only within limits. The "within limits" phrase can be understood in two different senses, as the law does encounter severe limits on each of the two independent variables involved in the paradigm.

First, Fitts' law can be explored only within narrow limits of relative amplitude—to this author's knowledge, it has never been possible in a standard Fitts' law experiment to have people perform an aimed-movement task whose *ID* would exceed 10 bits, that is, a *D*/*W* ratio of about 500 (Guiard, Beaudouin-Lafon, & Mottet, 1999; but see also Guiard, Bourgeois, Mottet, & Beaudouin-Lafon, 2000).

Second, practically speaking a Fitts task can be scaled up and down only to some extent. Were these scale limits known to be localized at some remote points above and below the selection of amplitudes used in typical Fitts' law experiments, the points raised in the present paper would be of immaterial importance. But the question of the actual localization of these limits on the scale continuum is essentially open, having been ignored so far, owing to the failure to isolate the scale factor in the classic AT approach. In fact, within *what* limits of scale Fitts' law remains valid is a problem that is experimentally intractable within the traditional AT approach. Yet there is a serious concern: so long as the effects of the scale factor are not understood and controlled in the paradigm, Fitts' law will amount to an optimistic guess.

Performance cannot be scale independent if only because perceptual-motor systems possess limited ranges of operation. For example, in a classic hand-tapping task, humans cannot cope with targets smaller than half a millimeter or so, owing to the limited resolution of their perceptual-motor system. This limited resolution prohibits scaling down an aimed-movement task below some critical level, unless participant are provided with a magnifying lens (Langolf, Chaffin, & Foulke, 1976; Guiard, Beaudouin-Lafon, & Mottet, 1999). On the other hand, any effector system, whether a single joint or the whole arm, also exhibits some upper limit for amplitude coverage.

From the mere recognition of the existence of a lower and an upper limit for the scale of any movement, it follows that performance should decline toward either end of the manageable range of scale. Thus, one may conjecture that in Fitts' paradigm the dependence of *MT* on scale exists and is U-shaped.

We now turn to the empirical evidence. We will first ask about the relationship that links *MT* to movement scale, considering some fresh data from an experiment that varied scale at a constant level of relative amplitude. We will then turn to the question of the functional relationship borne by relative and absolute amplitude, considering the data of two rare experiments of the literature that happened to manipulate these two factors orthogonally, in keeping with the ARA logic introduced in Section 2.5.

3.3. A U-Shaped Relationship Between MT and Movement Scale

Guiard and Slifkin (2001) investigated the effect of movement scale in a remote-controlled reciprocal Fitts task by manipulating the display-control (DC) gain over a large range with a constant *ID* of 5 bits. To make the screen cursor cover a constant *D* on a visual display, participants had, in different conditions, to move a slider with the hand over amplitudes of 1, 21, 41, or 61 cm, thanks to variations of the DC gain.

Reducing the DC gain means scaling up the absolute amplitude of movement at a constant level of relative amplitude, a case that was illustrated on the left-hand side of Figure 3. Notice that this scaling variation is restricted to hand space, to the exclusion of any change in the visual display—an interesting characteristic of DC gain manipulations. In particular, the amplitude of cursor motion on the display is not affected.

In addition to *MT*, we measured performance accuracy with an index of effective difficulty (ID_e) based on mean movement amplitude (*A*, rather than *D*) and effective tolerance (W_e, rather than *W*). Specifically, ID_e was computed as $\log_2(2A/W_e)$ (see Welford, 1968).

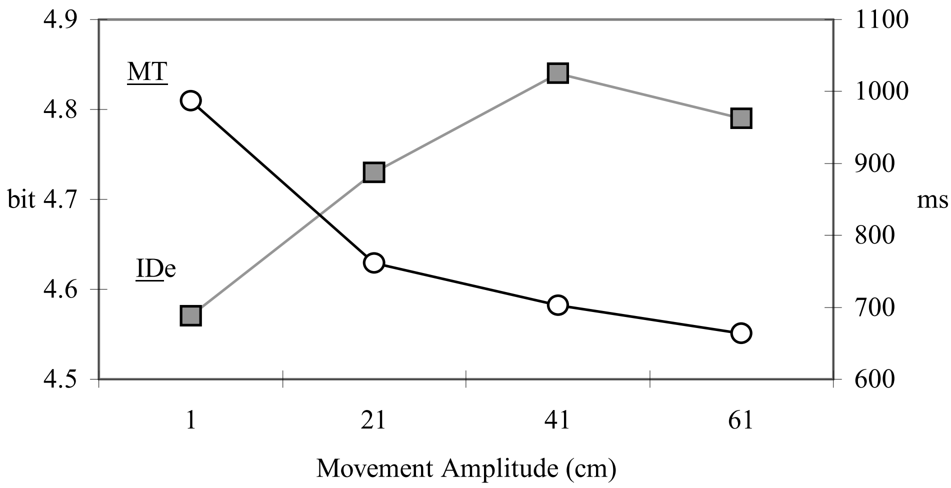

Figure 5 . Effect of movement scale on the speed (MT) and accuracy (ID_e) of movement in Guiard and Slifkin's (2001) reciprocal Fitts task experiment.

Scale had a consistent nonlinear influence on *MT*. As shown in Figure 5, scaling up the absolute amplitude of movement from 1 cm (with only the fingers involved) to 61 cm (with the whole arm involved) improved both movement accuracy ($F(3,33)=6.37$, $p<.002$) and movement speed ($F(3,33)=30.62$, $p<.001$).[8]

The hypothesis of a U-shaped relationship between *MT* and movement scale was supported by the data. As visible in the figure, movement precision began to drop beyond 41 cm, while a floor effect was simultaneously beginning to settle on *MT*, suggesting an optimal region for the movement around 40-60 cm.

This finding is consistent with the conclusion of a large body of studies conducted in an ergonomic perspective on the role of the DC-gain factor in remote-controlled tasks. This literature has produced converging evidence that the dependency of *MT* upon movement scale is U-shaped, and repeatedly confirmed the existence of scaling optima. For a given effector system in a given task, performance drops as soon as movement scale becomes either too small or too large, (e.g., Arnaut & Greenstein, 1990; Buck, 1980; Gibbs, 1962; Jenkins & Connor, 1949; Poulton, 1974; for a review, see Li Lin, Radwin, & Vanderheiden, 1992).

The simple suggestion that arises from the evidence reported and cited above is that it seems risky to ignore the scale factor in a Fitts experiment, scale being liable to exert a powerful non-linear effect on performance.

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[8] A full discussion of the specific *MT* curve we obtained, which differs considerably from some other observations—notably, those of Hoffmann (1997) and Langolf et al. (1976)—would take us away from the main point of this section.

3.4. The Functional Relationship Between Absolute and Relative Amplitude

Having examined separately the impact of scale on performance, we may turn to the question of how the paradigm's two independent variable interact, a question whose treatment requires an ARA factorial description of the paradigm. Unfortunately, the literature reports few experiments with an ARA design. Below we examine two rare cases, the studies of Gan and Hoffmann (1988) and Danion, Duarte, and Grosjean (1999).

Gan and Hoffmann (1988) studied the performance of discrete tapping movements at ten levels of *ID* (from 1 to 6 bits) for each of four levels of *D* (4, 9, 16, and 25 cm).[9] Figure 6 plots their *MT* data (from their Table 1, p. 832) as a function of relative amplitude (*D*/*W*), separately for each level of absolute amplitude (*D*). Interestingly, *MT* was linearly dependent on relative amplitude *D*/*W*. The scale factor was also found to substantially influence performance, with the movement slowing down monotonically as absolute amplitude was raised from 4 to 25 cm. Gan and Hoffmann reported a strong interaction between *D* and Fitts' logarithmic *ID*, but the Fitts' law curves they obtained were consistently nonlinear. In fact, as shown in Figure 6 (which uses a linear, rather than a log scale for relative amplitude), their data do suggest an essentially additive relation between relative and absolute amplitude.

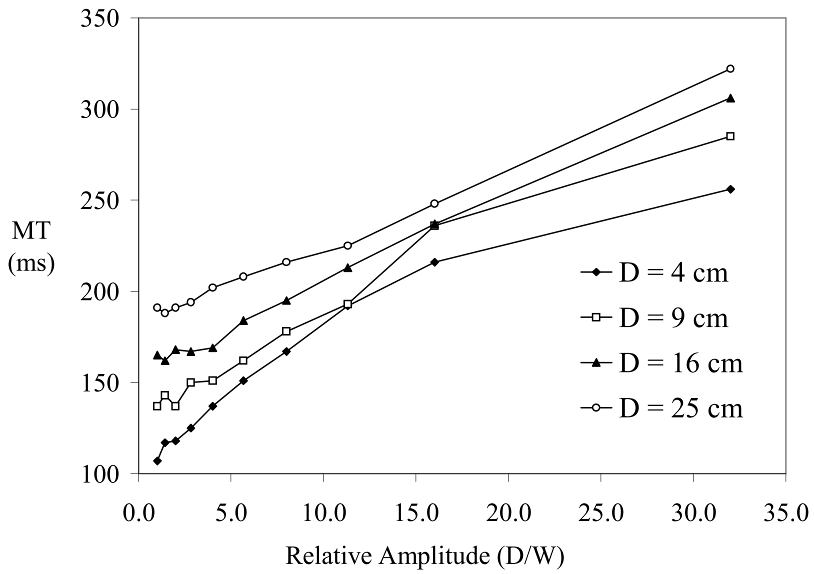


Figure 6. The MT data of Gan and Hoffmann (1988) re-plotted as a function of relative movement amplitude, at each of four levels of absolute amplitude.

Table 4. Goodness of fit (r^2) for the linear regression of *MT* versus task difficulty in the data of Gan and Hoffmann (1988), using three candidate definitions of the *ID*, for each level of scale.

[9] This work of Gan and Hoffmann (1988) was primarily aimed at empirically substantiating the distinction between ballistic and visually-controlled movement. Here we leave this concern aside to focus on the issue of absolute versus relative amplitude.

	D (cm)				
	4	9	16	25	Mean
Fitts' logarithmic $ID = \log_2(2D/W)$	.937	.838	.791	.754	.830
Meyer et al.'s (1990) power $ID = (D/W)^{1/2}$	.990	.965	.951	.926	.958
Linear $ID = D/W$	.923	.970	.994	.995	.971
Mean	.950	.924	.912	.892	

As shown in Table 4, a simple linear *ID* defined as D/W provides a better fit (with an r^2 of .971 on average over the four scale levels) than Fitts' logarithmic *ID* (r^2 = .830). The linear *ID* does also better than Meyer, Smith, Kornblum, Abrams, & Wright's (1990) square-root *ID* (r^2 = .958). Keeping in mind that the data set of Gan and Hoffmann (1988) is one of the few that permit the *MT* versus *ID* relationship to be evaluated without any spurious influence from scale, such a result is worthy of consideration.

Another instance of an utilization of the ARA approach to Fitts' paradigm is the recent study of Danion et al. (1999). [10] These authors, interested in the question whether Fitts' law holds for movements of the whole body, had participants stand on a force platform, facing a computer screen. The task was a reciprocal aiming task in which the screen cursor was made to move from one target to another by oscillating one's center of pressure on the platform. Danion et al. used six levels of task difficulty (*ID* = 1.4 through 2.9 bits, a low-level selection of *ID*s suitable to the particular effector system involved) crossed with four levels of movement scale on the platform (3, 4.5, 6, and 9 cm). Movement scale was varied by adjusting the DC gain.

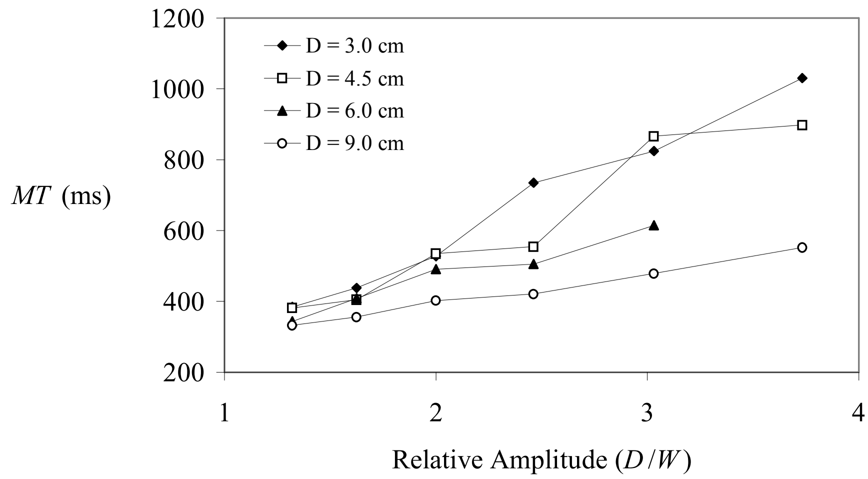


Figure 7. The data of Danion et al. (1999) with MT re-plotted as a function of relative movement amplitude, at each of four levels of absolute amplitude.

[10] The author thanks Frédéric Danion for making his numerical data available to him.

As shown in Figure 7, the results of Danion et al.'s whole-body pointing experiment differed from those of the hand-tapping experiment of Gan and Hoffman in two notable respects. First, performance improved, rather than decayed, as the movement was scaled up, a finding that presumably reflects the difficulty of controlling amplitudes of a few centimeters with oscillations of the whole body. Second, whereas in Gan and Hoffmann's experiment the effects of absolute and relative amplitude were essentially additive, Danion et al.'s data suggest a clear-cut interaction, with the effect of relative amplitude monotonically declining as the movement was scaled up.

Interestingly, however, the *MT* data of Figure 7 replicate those of Gan and Hoffmann (1988) in that they show an essentially linear link between performance and relative amplitude. Again, as summarized in Table 5, a better linear data fit obtains when task difficulty is quantified as D/W (mean $r^2 = .989$) than with Fitts' logarithmic *ID* ($r^2 = .947$) or Meyer et al.'s (1990) power *ID* ($r^2 = .961$).

Table 5. Quality of fit (r^2) for the linear regression of *MT* versus task difficulty in the data of Danion et al. (1999), for each level of scale, with three candidate definitions of task difficulty.

	Distance (cm)				
	3	4.5	6	9	Mean
Fitts' logarithmic $ID = \log_2(2D/W)$	.966	.902	.964	.957	.947
Meyer et al.'s (1990) power $ID = (D/W)$^1/2	.982	.920	.964	.978	.961
Linear $ID = D/W$	.987	.987	.958	.989	.980
Mean	.978	.936	.962	.975	

So the data of these two experiments converge to suggest the possibility of modeling Fitts' law as a linear equation—at least for limited ranges of variation of relative amplitude. Why in Gan and Hoffmann's study absolute and relative amplitude added their effects on *MT* while these factors interacted in Danion et al.'s is a question for future research.

These two isolated studies must be viewed just as a start. The functional relationship borne by absolute and relative amplitude in Fitts' paradigm is an important research problem that needs to be tackled, and this requires recourse to ARA designs like those of Gan and Hoffmann and Danion et al.

4. Relative Amplitude: Questioning the Difficulty Labeling

Designating as an "index of difficulty" some logarithmic or power transform of the ratio D/W has been a well-established tradition in the field since Fitts (1954). The previous sections of this paper used occasionally the term difficulty to refer to the paradigm's independent variable based on relative amplitude. However, as will be explained in this section, recourse, in the aimed-movement paradigm, to the very notion of difficulty is questionable. Three objections arise. First, difficulty is a subjective intermediate variable whose utility is uncertain. Second, the link between subjective difficulty and relative amplitude is not simple enough to justify the implicitly assumed equivalence. Finally, there seems to be no more affinity between difficulty and relative amplitude than between difficulty and absolute amplitude.

4.1. Difficulty: A Subjective Intermediate Variable

A first concern is that difficulty is a *subjective* variable. Sticking to operationally defined notions, the only thing we know for certain is the simple quantitative relationship that links *MT* to relative amplitude. Whether it is legitimate—and even useful, in the first place—to rephrase this relationship in terms of difficulty, a subjective variable that has the status of a hypothetical intermediate variable, as shown in Figure 8, seems questionable.

Figure 8. A direct and an indirect route of causality from relative amplitude to movement time. Difficulty plays the role of an intermediate subjective variable, interposed between two objectively defined variables.

This is not to suggest that the notion of difficulty is ill-defined in Fitts' paradigm. Undoubtedly, this notion is grounded on an unequivocally specified manipulation and an unequivocally specified dependent measure. Simply, the point is that when relative amplitude is manipulated and performance speed measured, reference to a third, intermediate variable like difficulty is gratuitous unless explicit justifications are provided. It is intuitively appealing to equate relative amplitude with difficulty, but Fitts' law literature has remained silent on the specific sense in which a movement can be said to become more difficult when its relative amplitude increases. We will see in the next two sub-sections that the needed justifications are rather elusive.

4.2. The Non-Linear Dependence of Difficulty on Relative Amplitude

A possible justification for equating relative amplitude with subjective difficulty could be found in the demonstration that these two variables bear a simple linear relationship. This, however, is unlikely. We will review evidence that the relationship is non-linear, with a minimum of subjective difficulty at some optimal region of relative amplitude.

One method of experimentally evaluating subjective difficulty in a Fitts task consists of examining the way in which participants actually comply with the accuracy instructions. It may be hypothesized that the more difficult a task subjectively, the greater the mismatch between effective difficulty (ID_e), which can be estimated on the basis of the distribution of the actual movement endpoints (e.g., as $\log_2(2A/W_e)$), and prescribed difficulty (ID_p), defined on the basis of the mere description of the experimental material (e.g., as $\log_2(2D/W)$). The relationship between subjective difficulty and relative amplitude can be inferred from the extent to which ID_e follows the variations of ID_p. For example, if a participant finds it difficult to handle a very high ratio of D/W, then the ID_e should be lower than the ID_p.

This question was investigated by Guiard and Ferrand (1998) in a remote-controlled, reciprocal aiming task. Their participants were explicitly asked to try, in all conditions, to use "*only* and *all* the error tolerance offered", while minimizing *MT*. The ID_e was found to vary linearly with the ID_p (y = 0.75 x + 1.49, r^2 = .994), but the slope was consistently less than unity, with a large positive intercept, as illustrated in Figure 9.

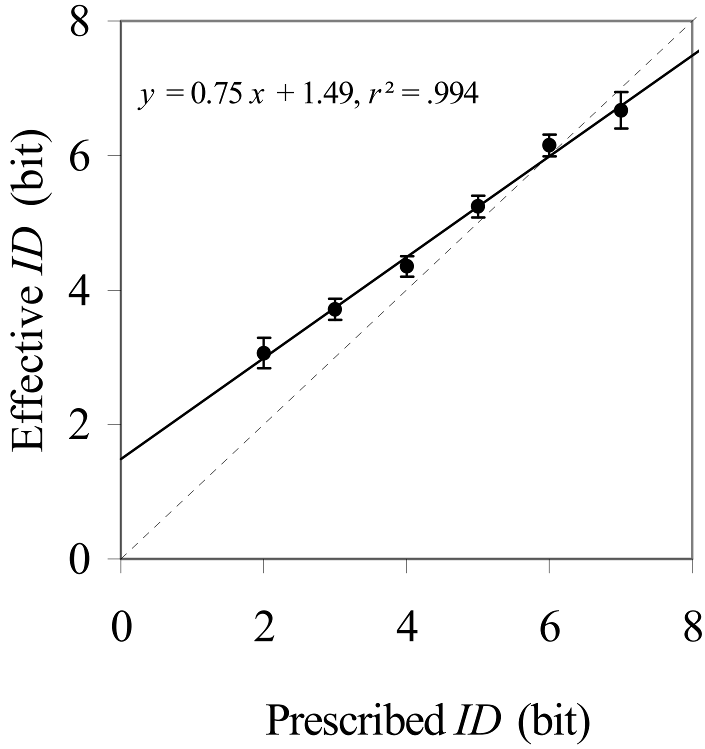


Figure 9. Graphical illustration of the way in which the ID_e follows the variations of the ID_p in the data of Guiard and Ferrand (1998). Error bars represent α = .05 confidence intervals based on between-participant standard deviations. The solid line and the dashed line illustrate the best-fitting curve and the line of equality, respectively.

In keeping with classic observations (e.g., Welford, 1968; Crossman & Goodeve, 1963/1983), the participants were less accurate than required when the ID_p exceeded some critical upper level. Below this level, however, ID_e was not found to equal ID_p. Rather, participants produced movements that were consistently more accurate than those required, and this mismatch was more and more marked as the task became less and less "difficult". Such a reluctance to execute an easier movement seems rather paradoxical, unless it is recognized that the supposedly less difficult task conditions are in fact, in some other sense, *more* difficult.

The solution to the paradox proposed by Guiard and Ferrand (1998) was that below some optimal region of the ratio D/W, the lower this ratio, the harder the task in terms of *energetic* demands. Think of an extremely easy Fitts task with, say, $D = W = 20$ cm (the ID amounting to a minimal 1 bit). Since the probability of a miss is virtually zero, the only concern that remains is to perform the movement as fast as possible. The point is that, owing to Schmidt et al.'s (1979) law, to try to exploit all the tolerance offered so as to match the typical instructions of a Fitts task, one needs to produce one's athletic maximum, and this, undoubtedly, is very difficult.

So we reach the conclusion that when humans are asked to carry out a speeded aimed-movement task, they are just as reluctant to deal with very low levels of difficulty as they are to deal with very high levels of difficulty, "difficulty" being understood here in Fitts' sense. While this seems rather paradoxical in light of Fitts' (1954) understanding of difficulty, the paradox vanishes altogether from the moment it is acknowledged that subjective difficulty involves at least *two* dimensions, energetic expenditure and information processing.

Since the energetic and the information processing costs vary in opposite directions when the ratio D/W changes, it is reasonable to hypothesize that for any aimed-movement task there must exist an optimal region of relative amplitude. In the data of Figure 9, this optimal region seems to have been located towards an ID of 6 bits. Only at this level of the prescribed ID were the participants able to comply with accuracy instructions—that is to say, to produce $A/W_e = D/W$.

If the link between relative movement amplitude and subjective difficulty is non-linear, then the legitimacy of the usual association of relative amplitude with difficulty seems quite questionable. Converting relative amplitude into the subjective variable of difficulty can only amount to importing, rather gratuitously, an extra dose of complexity into the paradigm.

4.3. The Dual Source of Difficulty in Fitts' Paradigm

A further problem with the equivalence that has been traditionally assumed between relative amplitude and difficulty is that there does no seem to exist any compelling affinity between these two variables. In fact, subjective difficulty is likely to be just as strongly dependent on the paradigm's other independent variable, movement scale.

Imagine you perform a Fitts' task that involves a medium level of absolute and relative amplitude. Now the experimenter scales up D and W proportionally over and over. Obviously, the task will become more and more difficult, up to the point where you will no longer be able to carry it out. The same will occur if the task is gradually scaled down below the optimal region.

We have seen in Section 3 that MT increases when an aimed movement is scaled up or down from some optimal scale region. If, as is the case with relative amplitude, MT is accepted as an objective criterion of difficulty, then it is clear that re-scaling a task above and below the task's optimal scale region implies an increase in task difficulty.

In sum, not only is the relation between relative amplitude and subjective difficulty too complex for any equivalence to hold, but a similar non-linear relation must be assumed to exist between subjective difficulty and scale, the paradigm's other independent variable. Therefore, it seems preferable to stick to operational definitions and to label the two independent variables as relative amplitude and absolute amplitude (or scale), rather than difficulty and scale.

5. The Deficiency of the Two-Dimensional Approach to Fitts' Paradigm

In this section, we proceed to evaluate the damage that has resulted in published work from traditionally overlooking the experimental confounds that affect variables D and W, as a result of the paradigm missing the third conceptual dimension of scale. We start with a taxonomic point: experiments with AT designs fall in three categories according to where the selection of Ds happens to fall on the scale continuum, relative to the effector-specific optimum. We then turn to an analysis of the shortcomings of the current method. One detrimental consequence of using AT designs has been the recurrent emergence of an ill-posed problem whose treatment has occasionally led to erroneous conclusions—the problem of the respective influences of D and W on performance. Another consequence, which we will document with two simulation experiments, has been the contamination of experimental measures by uncontrolled scale effects, leading to biased estimates of Fitts' law.

5.1 A Taxonomy of AT-Designed Experiments

What possible effects can an uncontrolled variation of movement scale exert on the assessment of Fitts' law in an experiment using an AT design? Three cases must be distinguished.

Insofar as *D* varies in the *near-optimal* region of the movement scale continuum (where the slope of the *MT* versus scale relationship is minimal, or even possibly null in case of a plateau), one expects little or no contamination from the scale factor. This, of course, is the only favorable case.

If, however, the selection of *D*s unfortunately extends in the *sub-optimal* scaling region (where scaling up the movement facilitates performance), the overlooked scale factor will counteract the effect of the *ID* and thus cause an underestimation of the slope of Fitts' law.

Finally, if the selected range of variation of *D* happens to extend in the *supra-optimal* region (where scaling up the movement deteriorates performance), scale will add its effect to that of the *ID*, leading to an overestimation of the slope of Fitts' law.

5.2. The Confound of Absolute and Relative Amplitude in AT Designs

The amplitude versus tolerance description of the aimed-movement paradigm was popularized by Fitts (1954; Fitts & Peterson, 1964). Table 6 shows the design he used for his famous 1954 study (Experiment 2), in which he crossed four levels of *D* with four levels of *W*. One expects *MT*s to be shown in a table of this sort, but notice that in fact Table 6 displays, for each combination of *D* and *W*, the levels taken by the third relevant experimental variable, namely, the *ID*.

Table 6. The *ID* as a function of target distance and target width in Fitts (1954, Experiment 2).

		D (cm)				
		10.2	20.3	40.6	81.3	Mean *ID* (bit)
	0.16	7.0	8.0	9.0	10.0	8.5
W (cm)	0.32	6.0	7.0	8.0	9.0	7.5
	0.64	5.0	6.0	7.0	8.0	6.5
	1.27	4.0	5.0	6.0	7.0	5.5
	Mean *ID* (bit)	5.5	6.5	7.5	8.5	7.0

The problem that is made apparent by this presentation is that variables *D* and *W*—two supposedly independent variables—each co-varied with the *ID*. As *D* was scaled up from 10.2 to 81.3 cm, the mean *ID* increased from 5.5 to 8.5 bits. Likewise, as *W* was scaled up from 0.16 to 1.27 cm, the mean *ID* declined from 8.5 to 5.5 bits. Thus each of the two variables Fitts used as factors in his experimental design co-varied with the *ID*.

It must be realized that the cost of a complete crossing of variables *D* and *W* is an incomplete crossing of variables *ID* and *D*. Table 7 shows the *MT* data that Fitts (1954) collected in his disc-transfer experiment.[11] Two observations must be made. First, the selection of *D* levels was systematically shifted upward as the *ID* increased—this being possible only because 12 of the 28 cells were left empty—and therefore the *ID* co-varied with target distance (a rather strong positive correlation, $r = .985$). Specifically, as task difficulty (top row) was manipulated from 4 to 10 bits, the mean amplitude of the movement (bottom row) increased from 10.2 to 81.6 cm—indeed a considerable variation.

Table 7. The *MT*s (s) of Fitts' (1954) disc-transfer experiment shown as a function of the paradigm's true independent variables, relative amplitude (*ID*) and absolute amplitude (*D*). Empty cells are filled in gray. The bottom row shows the mean value taken by *D* for each level of the *ID*.

		ID (bit)						
		4	5	6	7	8	9	10
D (cm)	10.2	0.535	0.607	0.649	0.697			
	20.3		0.623	0.672	0.734	0.771		
	40.6			0.724	0.771	0.844	0.896	
	81.3				0.902	0.975	1.028	1.096
	Mean *D*	10.2	15.2	23.7	38.1	47.4	61.0	81.3

The reason why such a level of co-variation is troublesome is because the overlooked variable influenced *MT* substantially. The table shows quite clearly that, considered separately for each level of task difficulty, Fitts' scaling up of *D* (together with *W*) resulted in the performance systematically slowing down. Thus, it is clear that each of the two confounded variables of Table 7 were actually influential in this experiment, a severe handicap for interpreting *MT*.

The data of Table 7 are illustrated graphically in Figure 10. Fitts' method of processing the data consisted of computing a linear regression with all data points.[12] The best-fitting curve Fitts obtained with this method, of equation $MT = 90\ ID + 150$, is shown in the

[11] See Sheridan (1979) for a similar criticism of the structure of Fitts' data.

[12] Fitts in fact originally introduced this method in a later reanalysis of his 1954 data (Fitts & Peterson, 1964).

figure as a solid line crossing the whole selection of *ID*s. This curve ought to summarize a pure effect of task difficulty, but it is in fact seriously contaminated by the variation of scale. Given the structure of the design, It would have been more satisfactory to compute a best-fitting curve *separately for each level of D*, to get estimates of Fitts' law free from any influence from scale, irrelevant here. These curves are shown as dashed lines in the figure. The revised method, however, yields rather different estimates of Fitts' law parameters with, in particular, considerably shallower slopes (52.8, 50.6, 58.9, and 63.5 ms/bit from the lowest to the highest levels of *D*) in comparison with Fitts' own estimate (90 ms/bit).

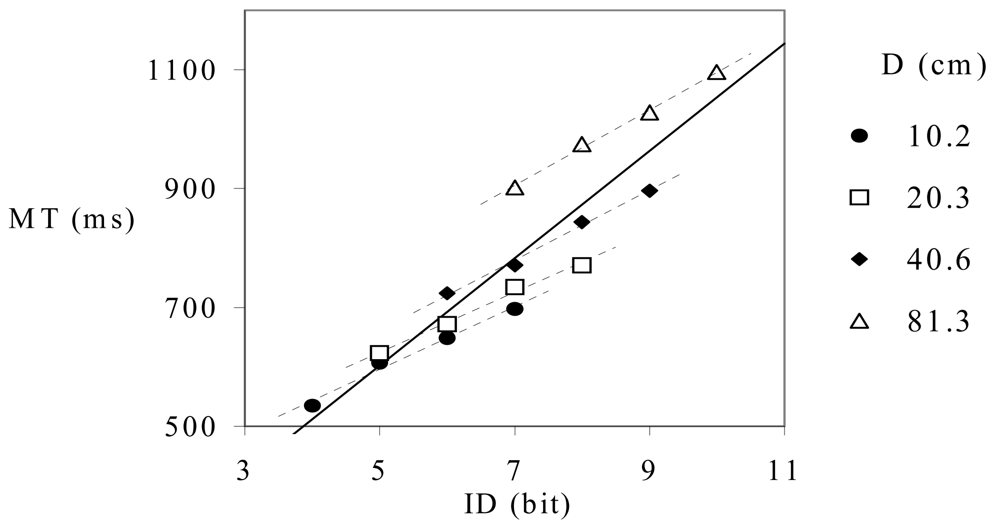


Figure 10. The MT data collected by Fitts (1954, Experiment 2) in his disc-transfer experiment.

In light of the analysis of Section 2, it is clear that the shortcoming of Fitts' design reflects his using only two dimensions, *MT* and relative amplitude, for thinking of his paradigm: with a single independent variable in the design, there is no way out of the factor confound.

The present criticism would just be of historic interest, had not Fitts' design been perpetuated up to present in mainstream research on human aimed movement. Ever since Fitts, it has been customary in the field to design experiments in which distance and tolerance are scrupulously balanced, as though these were independent variables, with the drawback that absolute and relative amplitude are made to co-vary. Presumably, hundreds of references could be cited here, including most leading contributions to the various chapters of Fitts' law research (to cite just a few: Accot & Zhai, 1997; Card, English, & Burr, 1978; Fitts &

Peterson, 1964; Jagacinski & Monk, 1985; Kelso, Southard, & Goodman, 1979; Meyer et al., 1990; Mottet & Bootsma, 1999; Welford, Norris, & Shock 1969).

Table 8. Estimation of the Co-Variation of Absolute and Relative Amplitude in a Sample of Fitts' Law Experiments [(*)]

	Absolute Amplitude (D)				Co-variation of D relative to $\log_2(2D/W)$	
	Unit	Min	Max	Range	r	Slope (% per bit)
Accot & Zhai (1997)	cm	12.8	51.2	38.4	.994	25.0%
Annett et al. (1979)	cm	20.3	40.6	20.3	.388	15.1%
Card et al. (1978)	cm	1.0	16.0	15.0	.815	13.9%
Fitts (1954) tapping	cm	5.1	40.6	35.6	.985	16.5%
Fitts (1954) disk transfer	cm	10.2	81.3	71.1	.985	16.5%
Fitts (1954) pin transfer	cm	2.5	40.6	38.1	.969	12.8%
Jagacinski & Monk (1985) helmet	deg	2.5	7.5	5.1	.996	31.0%
Jagacinski & Monk (1985) joystick	deg	2.5	7.5	5.1	.996	31.3%
Kelso, Southard, & Goodman (1979)	cm	6.0	24.0	18.0	.894	40.0%
MacKenzie & Buxton (1992)	pixel	64.0	512.0	448.0	.985	16.5%
Meyer et al. (1990)	deg	10.0	39.5	29.5	.993	24.9%
Mottet & Bootsma (1999)	cm	8.0	24.0	16.0	.747	20.5%
Welford et al. (1969)	cm	3.4	40.0	36.6	.709	20.5%

[(*)] *Note*. The rightmost estimate is the slope of the *D* vs. *ID* relationship expressed in percentage of the range of *D* covered in the experiment per bit. Fitts' *ID* was generalized to all data sets to facilitate comparisons.

Table 8 shows the strength of the undesirable correlation that linked absolute and relative amplitude in a sample of important studies from the literature. The correlation coefficients take very high positive values, in most cases over the .9 level. This seems alarming because the slope of the unwanted link between *D* and the *ID* was generally quite substantial, as shown in the rightmost column of the table. Take for example, the case of Meyer et al.'s (1990) influential study: for every new bit of information in the manipulation of relative amplitude, movement scale was increased on average by 25 percent of its total range of variation (7.5° per bit).

Two categories of exceptions can be cited: studies in which task difficulty was manipulated through variations of *W* at a constant level of *D* (e.g., Guiard, 1993, 1997; Mottet, Guiard, Bootsma, & Ferrand, in press) and studies whose experimental design fully crossed movement difficulty and movement scale (Danion et al., 1999; Gan & Hoffmann, 1988). We will return to these studies below.

With some examples, the next sections will show that failure to detect the confound between absolute and relative amplitude with the traditional AT design has caused serious misunderstandings and measurement errors in Fitts' law research.

5.3 A Recurrent Ill-Posed Problem: Separating the Effects of D and W

One consequence of the established 2D understanding of Fitts' paradigm has been the recurrent reappearance of the ill-posed experimental problem of estimating the respective effects of variables *D* and *W* on *MT* (e.g., Annett, Annett, Hudson, & Turner, 1979; Card, English, & Burr, 1978; Jagacinski & Monk, 1985; Meyer et al., 1990; Mottet & Bootsma, 1999; Welford, Norris, & Shock, 1969).

According to Fitts' law, the two possible methods of manipulating the *ID*—by varying either the numerator or the denominator of the ratio *D*/*W*—are equivalent and hence they should yield the same coefficients of Fitts' law. However, the two methods have been generally reported to yield substantially different estimates.

Let us focus on one representative example. Welford, Norris, & Shock (1969), who used a reciprocal tapping task, were puzzled to find a Fitts' law slope of over 170 ms/bit through the variation of *W* (with *D* kept constant), but of hardly 100 ms/bit through the variation of *D* (with *W* kept constant). This apparent discrepancy led Welford et al. to the wrong hypothesis that " the accuracy of ballistic movements is, other things being equal, independent of their extent" (p. 11). This hypothesis—which must be understood to refer to absolute lengths, that is, to *W* rather than *W*/*D* and to *D* rather than *D*/*W*—has been clearly ruled out since then. For example, Schmidt et al. (1979) showed that, for very fast, purely ballistic aimed movements, the spatial dispersion of endpoints increases when, for a constant *MT*, movement amplitude is increased, and Langolf, Chaffin, & Foulke (1979) showed with kinematic analyses that the whole movement, and not simply its terminal component, is affected by the tolerance.

In light of the alternative ARA factorial description, the puzzle Welford et al. thought they had to tackle vanishes. As shown in the right-hand side of Figure 3, the *W* manipulation involves just relative amplitude (*D*/*W*), to the exclusion of scale (*D*), and so this method of estimating Fitts' law is valid. By contrast, the separate manipulation of *D*, with *W* constant, varies both relative and absolute amplitude (*D*/*W* and *D*), thereby exposing *MT* to their confounded influences. From the moment it is realized that one of two concurrent methods of evaluating an effect is invalid, there is no more reason to wonder about discrepant results.

But we can say more, with the ARA approach, about these results. We can explain why Welford et al. obtained a shallower slope for Fitts' law with the separate manipulation of *D*, rather than *W*. Their data show that scaling up the movement facilitated performance systematically over the whole selection of absolute amplitudes that was used in the experiment (34-400 mm), revealing a sub-optimal selection of task scalings. This scale effect accounts for the reduced slope of Fitts' law with the manipulation of *D*: as the increase of relative amplitude made performance more difficult, the simultaneous scaling up of absolute amplitude made the performance, in another sense, less difficult. So the uncontrolled effect of scale involved in the isolated manipulation of *D* partially *offset* the effect of the *ID*.

In fact, the problem of the respective contributions of *D* and *W* to Fitts' law is squarely intractable because it is ill-posed, the distance and tolerance concepts being inherently equivocal in the usual 2D understanding of the paradigm. To investigate the effect of absolute amplitude or absolute target size, one needs to distinguish these variables from relative amplitude and relative target size, but that requires the problem to be rephrased in the ARA conceptual space, with the third dimension of scale.

5.4. Evaluating the Scale Bias Inherent in the Usual AT Design: Two Simulation Experiments

In designing a Fitts' law experiment, one needs more cells to cross absolute and relative amplitude (ARA design) than amplitude and tolerance (AT design). So by re-processing the data of an experiments with a full ARA design, it is possible to simulate an experiment with an AT design and thus evaluate the results that would have obtained, had the traditional AT design been used. Re-computing Fitts' law from a subset of the data such that *D* is crossed with *W* is like re-running the experiment with an AT design. The question, of course, is, how different would have the results been? Such a simulation was performed on the data of Danion et al. (1999) and Gan and Hoffmann (1988), the two already cited instances of ARA-designed Fitts' law studies. It will be shown that AT designs provide quantitatively biased estimates of Fitts' law and that, worse, they can be misleading with regard to the issue of how the law should be modeled qualitatively.

Quantitatively Biased Coefficient Estimates

Danion et al.'s ARA design, which crossed six levels of *ID* with four levels of *D*, had 24 cells. From these 24 cells, a subset of nine cells could be selected for the simulation in such a way that three levels of *D* were crossed with three levels of *W* (see Table 9).

Interestingly, the range of variation of the *ID* was not altered in the simulation, with the *ID* still varying from 1.4 to 2.9 bits.

Recall that the main problem with the AT design is that it produces an orthogonal variation of *D* and *W*, of limited utility, at the considerable cost of introducing a factorial confound between relative and absolute amplitude. In the simulated AT experiment based on Danion et al.'s data, the correlation between *D* and *W* was found indeed to be very low and statistically non significant ($r = .135$), and this led to a strong positive correlation between *D* and the ratio *D*/*W* ($r = .835$, $p < .001$).

Table 9. Combinations of *D* and *W*, along with the *MT*s, for the largest possible subset of cells from Danion et al.'s (1999) experiment such that *D* varies orthogonally with *W*. From the original 24 cells, the selection retains three levels of *D* crossed with three levels of *W*.

D (cm)	*W* (cm)	D/W	Fitts' *ID* (bit)	*MT* (s)
4.50	3.41	1.32	1.40	0.382
4.50	2.77	1.62	1.70	0.405
4.50	2.25	2.00	2.00	0.535
6.00	3.69	1.62	1.70	0.408
6.00	3.00	2.00	2.00	0.491
6.00	2.44	2.46	2.30	0.505
9.00	3.66	2.46	2.30	0.421
9.00	2.97	3.03	2.60	0.478
9.00	2.41	3.73	2.90	0.552

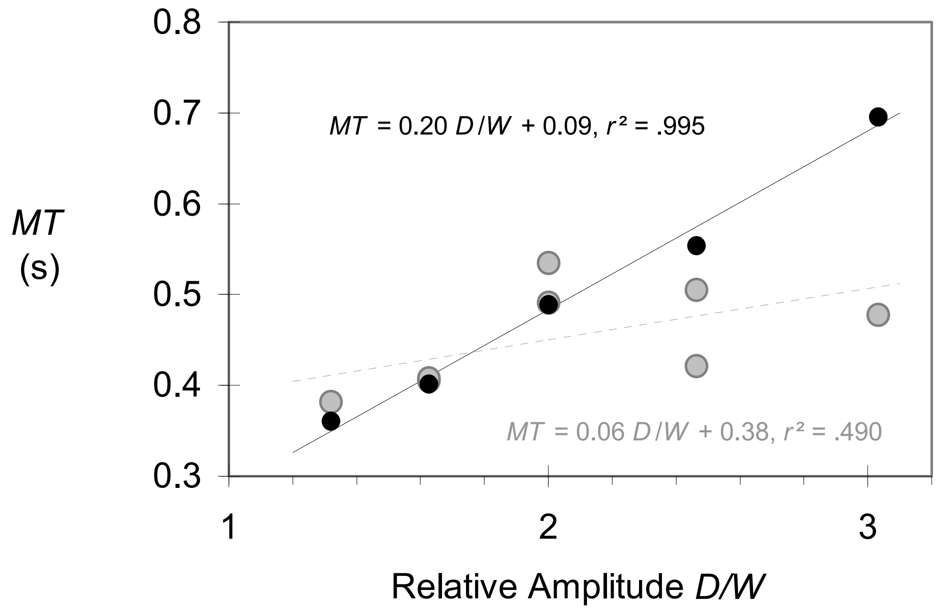


Figure 11. A simulation of the results Danion et al. (1999) would have obtained, had they used the usual AT design for the same range of *ID*s. The simulation data are illustrated in gray, while the data they actually obtained with their ARA design, averaged over the four scale levels, are illustrated in black.

As visible in Figure 11, the simulated AT experiment yielded a picture of Fitts' law that differs markedly from that which Danion et al. actually obtained with their ARA design. Most notably, the slope of the fitted function for the simulated data is hardly one third of that obtained with their actual data.

The interpretation of such a slope reduction with the simulated AT design is straightforward. Danion et al.'s actual data (see Figure 7) show that scaling up the task improved performance monotonically—the symptom of a sub-optimal scale selection. Owing to the positive correlation of absolute and relative amplitude inherent in the simulated AT design, the effect of scale (the larger the absolute amplitude, the better) partially offset the effect of relative amplitude (the larger the ratio D/W, the worse), and so the AT simulation produced an underestimation of the slope of Fitts' law.

Table 10. Movement times for the largest subset of cells from Gan and Hoffmann's (1988) design such that *D* and *W* vary orthogonally.

D (cm)	*W* (cm)	D/W	Fitts' *ID* (bit)	*MT* (s)
4.0	4.00	1.00	1.0	0.107
4.0	2.83	1.41	1.5	0.117
4.0	2.00	2.00	2.0	0.118
4.0	1.41	2.83	2.5	0.125
4.0	1.00	4.00	3.0	0.137
9.0	4.50	2.00	2.0	0.137
9.0	3.18	2.83	2.5	0.15
9.0	2.25	4.00	3.0	0.151
9.0	1.59	5.66	3.5	0.162
9.0	1.13	8.00	4.0	0.178
16.0	4.00	4.00	3.0	0.169
16.0	2.83	5.66	3.5	0.184
16.0	2.00	8.00	4.0	0.195
16.0	1.41	11.31	4.5	0.213
16.0	1.00	16.00	5.0	0.237
25.0	4.42	5.66	3.5	0.208
25.0	3.13	8.00	4.0	0.216
25.0	2.21	11.31	4.5	0.225
25.0	1.56	16.00	5.0	0.248
25.0	0.78	32.00	6.0	0.322

Gan and Hoffmann used a copious ARA design for their experiment, crossing ten levels of *ID* with four levels of *D*. From the 40 cells of this design, the simulation used a subset of 20 cells such that four levels of *D* were crossed with five levels of *W* (see Table 10). As was the case with Danion et al.'s data, the AT simulation let the *ID* cover the same range of variation as in the original ARA experiment (1-6 bits). The simulation produced a virtually zero correlation between D and W (r = .02) and, by the same token, the strong positive correlation between D and D/W expected with an AT-designed experiment ($r = .665$, $p < .01$).

The results of the simulation are shown in Figure 12. The 40% inflation of the slope with the AT simulation (from 50 to 70 ms per unit of D/W) can be easily explained. Gan and Hoffmann, contrary to Danion et al., obtained a monotonic increase of MT as they scaled up the aimed-movement task (see Figure 6), revealing a supra-optimal selection of scale. Hence, the AT simulation made absolute and relative amplitude work in synergy, leading to an overestimation of the effect of the ID.

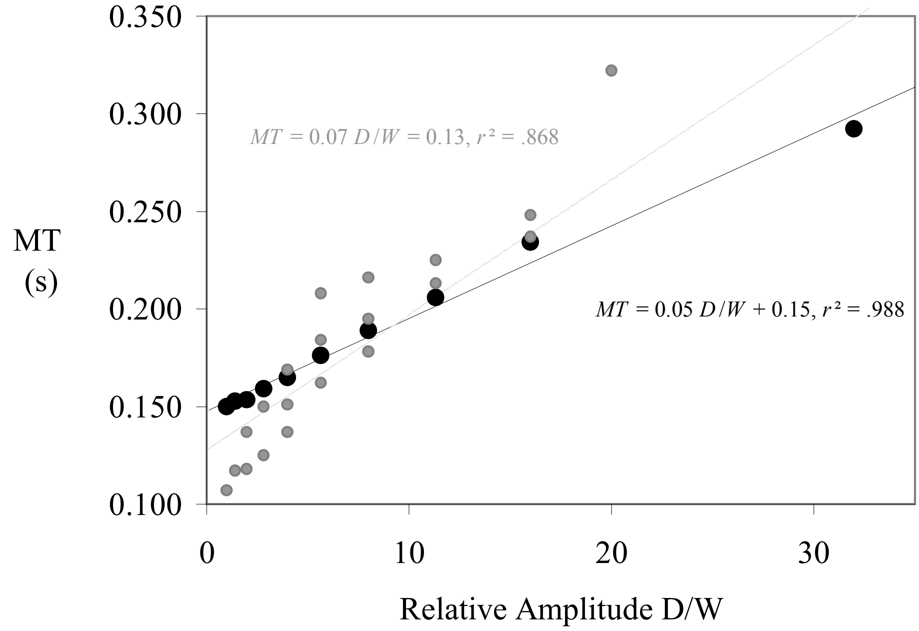


Figure 12. A simulation of the results that Gan and Hoffmann (1988) would have obtained, had they used an AT design for the same range of IDs. The simulation data are shown in gray. The data they actually obtained with their ARA design, averaged over the four levels of D, are shown in black.

Failure to Identify the Best-Fitting Model

Table 11 summarizes the results of the two simulation experiments. We have just seen that within one and the same model of Fitts' law, the ARA and the AT designs yield different coefficients for the same experimental conditions. This confirms that the AT design does indeed induce a quantitative bias in the assessment of Fitts' law (in the left part of Table 11, Fitts' logarithmic model is just taken as an example).

Table 11. Comparison of the actual ARA and the simulated AT experiments of Danion et al. (1999) and Gan and Hoffmann (1988) in terms of Fitts' law coefficients and quality of fit with four candidate models (*).

	Fitts' Law Coefficients using Fitts' (1954) *ID*		Mean Fit (r^2) with Four Candidate Models			
	Slope (s/bit)	Intercept (s)	Fitts' *ID* $\log_2(2D/W)$	Shannon's *ID* $\log_2(D/W+1)$	Power *ID* (D/W)^0.5	Linear *ID* D/W
Danion et al. (1999)						
Actual ARA Design	0.293	-0.082	.947	.957	.961	.980
Simulated AT Design	0.095	0.265	.529	.517	.511	.490
Gan & Hoffman (1988)						
Actual ARA Design	0.026	0.101	.830	.878	.958	.971
Simulated AT Design	0.040	0.043	.895	.937	.937	.868

(*) *Note*. For the ARA design, the slope, intercept, and r^2 levels are averages over four scale levels. Shannon's *ID* is that advocated by MacKenzie (1992), and the power *ID* is that advocated by Meyer et al. (1988).

But recourse to the AT logic has a yet more serious consequence. As shown in the right-hand side of Table 11, the best-fitting equations rank differently depending on whether the AT or the ARA design is used to assess Fitts' law. With an AT design, Danion et al. would have obtained deceptively low levels of r^2 for all candidate models (the highest r^2 being a modest .529, for Fitts' model), and perhaps they would have hesitated to claim that Fitts' law holds for postural oscillations of the whole body. In fact, with their actual ARA design, they found good fits, with r^2 = .947 for Fitts' equation. However, it turns out that their data are best modeled by a linear equation, of the form $MT = k_1 + k_2 * D/W$, as visible in the table. Had Danion et al. had recourse to the usual AT design, their data would have failed to reveal the possibility of modeling Fitts' law as a linear, rather than logarithmic equation.

Had Gan and Hoffmann used an AT design and asked about the best fitting model, they would have been led to conclude that Shannon's logarithmic model (MacKenzie, 1992) and Meyer et al.'s (1990) power model did best, and that the linear model did worst, but this would have been a false conclusion. In fact, with the data they actually obtained with their ARA design, the linear model does best, and Fitts' does worst.[13] So the reprocessing of the two data sets suggests that the bad consequences of using the traditional design extends beyond the issue of coefficient estimation—in fact, the AT design is liable to mislead Fitts' researchers as to which mathematical model is the most consistent with their data.

Were the alternative equations of Fitts' law just convenient ways of summarizing empirical data, the above findings would be relatively harmless. However, these mathematical formulations actually reflect specific theories of movement control. Consider Meyer et al.'s (1990) theory. The power form of the equation they proposed was based on an elaborate

[13] The reports of Danion et al. (1999) and Gan and Hoffman (1988), who used Fitts' *ID*, do not consider the issue of the logarithmic versus power versus linear *ID*.

model, the stochastic optimized sub-movement model. Meyer et al. hypothesized that an aimed movement is made up of an optimized number of concatenated sub-movements, each of which is stochastically optimized to accommodate the fact that the faster the sub-movement, the greater the motor noise (Schmidt et al., 1979) and hence the more likely the necessity to produce an additional corrective sub-movement when the current sub-movement is completed. The empirical validation of Meyer et al.'s (1990) model was based on the joint consideration of both chronometrical (*MT* measures) and kinematic (sub-movement parsing) aspects of performance. It would take us beyond the scope of the present paper, concerned by the conceptual framework of the study of aimed movement rather than the theory of movement control, to discuss Meyer et al.'s model. What should be noted at this point is simply that since the design of Meyer et al.'s experiment involved a high degree of co-variation between relative and absolute amplitude (see Table 8 and Section 5.2), our findings raise doubts about the empirical grounding of their conclusions and suggests the necessity of new experimental tests using an appropriate ARA design.

6. *Implications for Future Research*

The new understanding of Fitts' aimed-movement paradigm proposed in this paper inscribes itself in a larger and, it is argued, clearer conceptual space whose three unequivocally defined dimensions are movement time and the absolute and relative amplitude of movements. This proposal has three important implications for future research. First, the experimental method of assessing Fitts' law can be improved by recourse to the ARA design, which makes it possible to eliminate the confounds that have handicapped Fitts' law research up to present. The second, deeper implication is that the theme of human aimed movement can be recast so as to progress towards a complete 3D account of human aimed-movement performance. Third, consideration of the scale dimension makes it possible to envision a promising adaptation of the paradigm to multi-scale pointing, a new research problem for human movement science that has recently emerged from the rapidly developing field of information technology.

6.1. Improving Fitts' Law Method Within the Classic 2D Approach

It was shown in Section 2 that Fits' aimed-movement paradigm has two irreducible independent variables, and that these are absolute amplitude (or movement scale) and relative amplitude. It was then shown, in Section 3, that both dimensions count: scale, the paradigm's dimension that has been generally ignored in Fitts' law studies, is a more influential source of variance for *MT* than has been thought so far, and therefore it cannot be legitimately ignored. In Section 4, it was argued that relative amplitude, the dimension that has attracted all the field's attention, is not more closely related to subjective difficulty than is scale, leading to the conclusion that it is safe, contrary to an established tradition, to ignore the variable of subjective difficulty and to stick to the simple operational notions of absolute and relative amplitude.

There is no reason, from the preceding, to question the importance of the classic problem inherited from Fitts, the problem of the two-dimensional relationship that links *MT* to relative amplitude. As has been noted many times, Fitts' law is a remarkably general and robust relationship—within limits of absolute and relative amplitude. The implication that has to be drawn, particularly from the analysis of the shortcomings of the traditional AT design (see Section 5), is that the traditional method of assessing Fitts' law can be improved.

The problem encountered in the classic study of Fitts' law is that the conventional AT experimental design fails to *neutralize* movement scale, a factor that the 2D approach, by definition, wants to ignore. The analyses of the above sections suggest that to adequately investigate the *MT* versus relative amplitude relation in 2D space, one needs to use ARA designs. This means that one needs, if not to theorize, at least to design one's experiments in the full 3D space of the paradigm. In practice, this simply involves filling up the design shown in Table 7, so as to make sure that the variation of the *ID*, the critical experimental variable, is orthogonal to the unwanted variation of *D*, a variable deemed to be irrelevant.

6.2. Tackling The Aimed-Movement Problem as a Whole: From Curves to Manifolds

The perspective outlined in the last sub-section seems too conservative. Let us try to see what it would mean to treat scale as a fully fledged experimental factor rather than as just a source of experimental perturbation—that is, as a factor to manipulate rather than a factor to neutralize.

One simple argument in favor of this more ambitious exploitation of the full 3D approach to Fitts' aimed-movement paradigm is that, despite the beauty of the scale invariance postulated by Fitts' law, performance is more often than not scale dependent, as emphasized in Section 3. Unless experiments are deliberately designed so as to restrict the range of the scale variation to virtually nothing, the absolute-amplitude variable is in its own right just as important in the paradigm as is relative amplitude.

Furthermore, if the goal of research on human aimed movement is to make reliable performance predictions, then building 3D models of aimed movement, of the form *MT* = f (*D*, *D*/*W*) rather than *MT* = f (*D*/*W*), should represent a progress towards that goal.

Finally, there is a practical argument. Thanks to the new computer software technology we now have at our disposal, the complexity cost of introducing a third dimension into the treatment of the aimed-movement problem is becoming manageable. An ever increasing variety of computer applications has been developing, thanks to which 3D patterns of data can be visualized and explored more and more easily. In this section we return to the data of Gan and Hoffmann (1988) and Danion et al. (1999), the two already mentioned instances of studies with ARA designs, to suggest that mastering data in 3D space is no longer a task of deterring difficulty.

What follows simply aims at helping to visually grasp the 3D conceptual space of Fitts' paradigm—fortunately, we have to deal with no more than three dimensions. Figures 13 and 14 show 3D representations of the data of Gan and Hoffmann (1988), which we have already examined in some details. However, while in the preceding figures we used 2D representations, with *MT* plotted as a function of relative amplitude and with scale treated simply as a parameter, now we can see the paradigm's full 3D space, with its three axes (*MT*, *D*, and *D*/*W*) and its three planes.

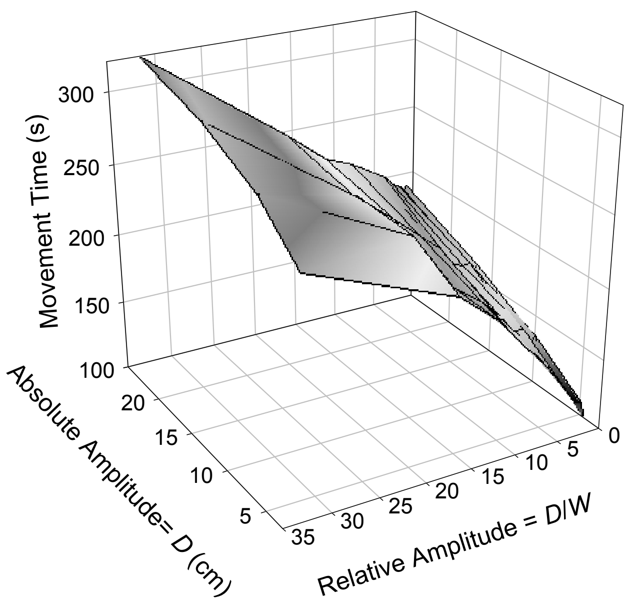
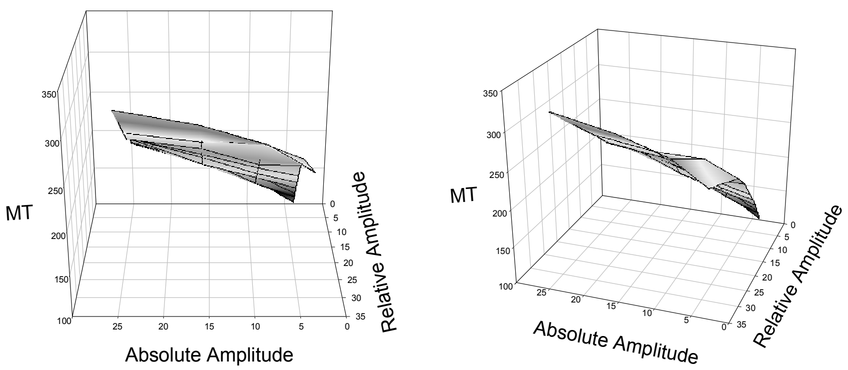


Figure 13. A 3D graphical representation of the data of Gan and Hoffmann (1988).

The floor of Figure 13 is the D x D/W plane, that in which we found the classic AT method to suffer a flaw (see Section 5). The problem with the AT design is that absolute and relative amplitude are not fully crossed, with the distribution of experimental points being typically far from rectangular, yielding a non-zero correlation between the two independent variables. In contrast, it is easy to see in Figure 13 that a vertical projection, onto the graph's floor, of Gan and Hoffmann's experimental data points would yield a rectangular scatter of data points, as a result of the ARA design used by these authors. The second plane (left-hand side wall) is the plane of Fitts' law, formed by the dimensions of relative amplitude and *MT*. With a horizontal projection of the data points on this plane, one would obtain the representation already shown in Figure 6. The third plane (right-hand side wall), is that formed by the dimensions of *MT* and scale, which so far has attracted more attention from an applied than theoretical perspective, the main reference here being the ergonomic literature on the effects of DC gain (see Arnaut & Greenstein, 1990).

The 3D representation of the data makes it possible to examine, not only each of the three 2D relations, but the whole 3D pattern. In the place of three 2D curves, one is faced by a single 3D object, which has the form of a manifold. With the help of the multiple viewpoint provided in Figure 14, the reader should be able to see that the data of Gan and Hoffmann (1988) make up an almost flat sheet—the 3D graphical image of a roughly additive combination of the main effects of absolute and relative amplitude.



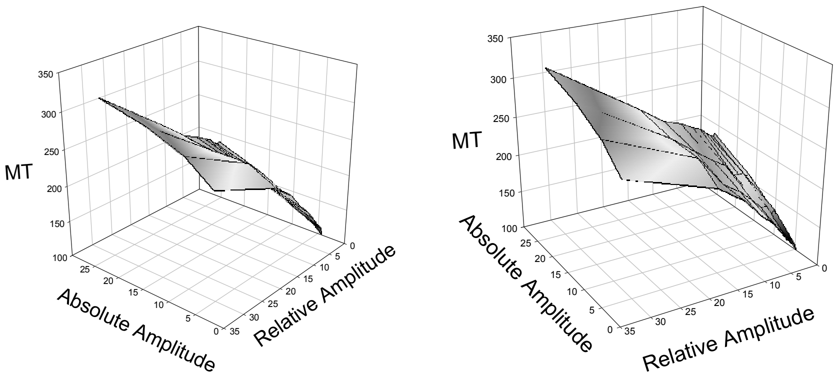


Figure 14. Four slightly different views on the data of Gan and Hoffmann (1988), to help to see the 3D shape of the manifold.

Figures 15 and 16 illustrate the data of Danion et al.' study, our second example of an ARA-designed study, whose layout is strikingly different. Unlike the almost planar sheet formed by the data of Gan and Hoffmann, the figures depict a deeply undulated manifold—the 3D image of the complicated sui generis interaction obtained by Danion et al. It is easy to appreciate that, in cases like this one, a 3D representation conveys more information than a flat representation like that shown in Figure 7, which simply shows a set of curves.

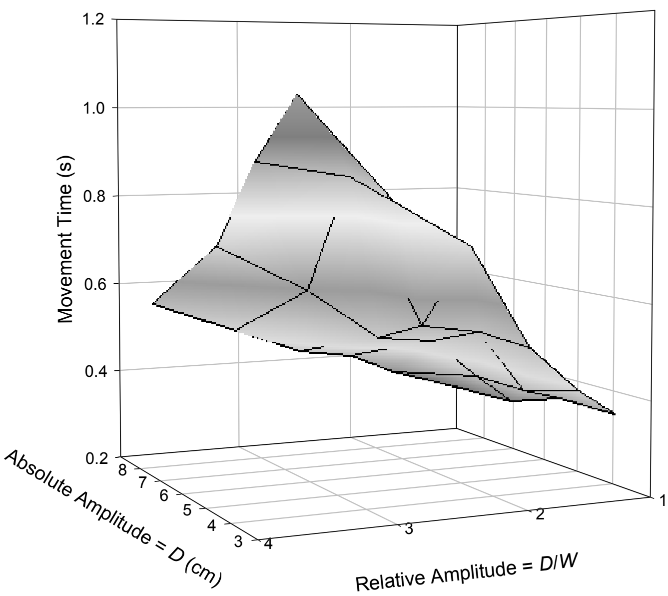


Figure 15. A 3D graphical representation of the data of Danion et al. (1999).

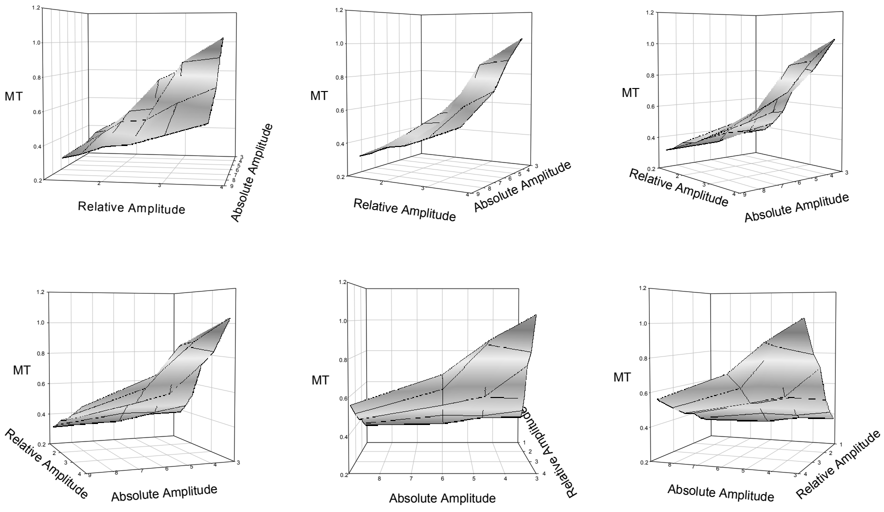


Figure 16. Six slightly different views on the data of Danion et al. (1999), to help appreciate the shape of the manifold.

6.3. A New Challenge for Aimed-Movement Research: Target Acquisition in Multi-Scale Electronic Worlds

This section introduces a recent extension of Fitts' aimed-movement paradigm that makes it possible to accommodate the problem of human aimed movement to the new context created by recent developments of information technologies. It will be shown that when aimed movement takes place in a multi-scale, or zooming graphical user interface, the narrow limits of absolute and relative amplitude that normally affect Fitts' law, due to the functional limitations of the human perceptual-motor system (see Section 3.2), vanish altogether. First, the fact that the scale of action becomes a freely-controlled parameter means, by definition, that the scale barrier to Fitts' law has fallen down. Second, and more intriguingly, we will see that with zooms any upper limit of relative amplitude disappears. This in turn makes it possible to investigate Fitts' law far above its usual upper limit of *ID*, a possibility that was recently explored by Guiard, Bourgeois, Mottet, and Beaudouin-Lafon (2000).

Dramatic changes have occurred recently in human-computer interaction (HCI), the freshly emerged branch of computer science specialized in the design of human-computer interfaces (see Shneiderman, 1998, for a review). These changes challenge Fitts' aimed-movement paradigm. The advent, during the nineties, of multi-scale or zooming graphical user interfaces (Furnas & Bederson, 1995), gave birth to *multi-scale* pointing, an entirely novel category of human action that has no counterpart in the real world (Guiard, 1999). In a zooming interface, not only can one move a cursor (an analogue to hand motion), not only can one move one's view (an analogue to locomotion), but one can freely adjust the *scale* at which one wishes to interact with the electronic world at hand. Note that a multi-scale world allows no absolute definition of variables *D* and *W*, these quantities being zoom dependent. The only independent variable that remains under the experimenter's control is the critical, zoom-independent *D*/*W* ratio—with the consequence that the above-discussed ARA versus AT design problem no longer exists.

Suppose you are exploring the zoomable maps of a recent electronic world atlas. For example, you are watching a city map of Wellington, New Zealand, but you decide to look for another, remotely located place, say, the Luxembourg garden in Paris, at a distance of over 19,000 km. You will first zoom out to see a large-scale view of the Earth, then perform a rapid pan[14] across the seas and the continents so as to position your view over Europe, and finally plummet down to France and then Paris by performing an intricate sequence of zooms in and pans. When, in the end, you are facing a detailed map of Paris, you will be able to click on the Luxembourg garden (perhaps to know the opening hours of this public garden, if your application includes this facility). The whole sequence, down to the final click on the target, will have lasted just a few seconds.

Not only does the above task belong unequivocally to the class of target-acquisition tasks to the study of which Fitts' paradigm is dedicated, but this task is perfectly well defined, with D = 19,000 km, W = 0.4 km, and therefore ID = 16.5 bits. Handling this high a level of *ID* would be impossible in the real world,[15] but it is a commonplace task for the user of a multi-scale electronic world.[16] As already noted, Fitts' law students so far have used ranges of *ID* that never extend beyond 10 bits (a *D*/*W* ratio of about 500), because so long as an aimed movement is confined in a single scale—the fate of real-world movements—one encounters the tight functional limitations of the human perceptual-motor system.

By contrast, as one explores a multi-scale electronic world, one can deal with however large values of *D* and however small values of *W* one likes, thanks to the zoom. Even though, of course, the current state of the technology and the amount of stored information impose ultimate limits to the *D*/*W* ratio, these limits are already extremely remote in comparison with those experienced in our real-world aimed movements.

[14] In the cinema metaphor that has become conventional in HCI research on multi-scale information visualization, zooming means re-scaling one's view and panning (a term derived from panoramic) means moving one's view in space.

[15] Of course, we *do* cover this sort of distances in the real world, but this requires concatenating a series of transportation acts (e.g., a walk + a taxi ride + a walk + a flight, etc.), none of which, it must be realized, obeys Fitts' law.

[16] A related example is selection of one page from among the 5,000,000 pages accessible on the Internet site of the US Library of Congress (Shneiderman, 1997), which can be said to involve an *ID* of 22 bits.

It is a new and important fact for Fitts' law research that humans are actually able to accommodate quite smoothly *ID*s far higher than 10 bits in target-acquisition tasks, provided that the user-environment interaction is zoomable (Guiard et al., 1999). The important question that immediately arises is whether Fitts' law holds *in general*, far beyond the so far inviolable barrier of *ID*. This question was recently addressed by Guiard, Bourgeois, Mottet, & Beaudouin-Lafon (2000), who used a zoomable pointing interface and thus were able to have their participants deal with *ID*s of up to 31 bits (i.e., up to a D/W ratio of one billion)—note that pointing with this high an *ID* was equivalent to reaching and selecting a one-inch target at a distance of about half the circumference of our planet.

Guiard et al.'s (2000) participants were offered three df of control, two for moving, with a mouse, the latitude and longitude of their view and a third one for zooming, using the throttle of a game controller assigned to the other, non-preferred hand. For *ID* levels up to 8 bits, the zoom facility was ignored, the participants being content to carry out the task by simply panning with the mouse. For higher *ID*s, all participants used the zoom for all their movements. The more difficult the task, the larger the zoom detour made by participants. In fact, the relation between zooming amplitude (A_Z)—the zoom range covered during the movement—and the *ID* was found to be rigorously linear, with $A_Z = 6.6\ ID - 14.8$, $r^2 = .997$.

Figure 17 shows the *MT* vs. *ID* curve obtained by Guiard et al., with the *ID* simply defined as $\log_2 (D/W)$. The *MT* was found to vary over the wide-ranging selection of *ID*s just as predicted by Fitts, with on average $MT = 0.32\ ID - 0.04$ ($r^2 = .995$), the fit being excellent for each of the seven individuals who participated in the experiment ($.981 < r^2 < .999$). Moreover, in keeping with Fitts' (1954) initial expectation of a constant rate of information transmission in humans, the intercept of the mean curve was virtually zero.

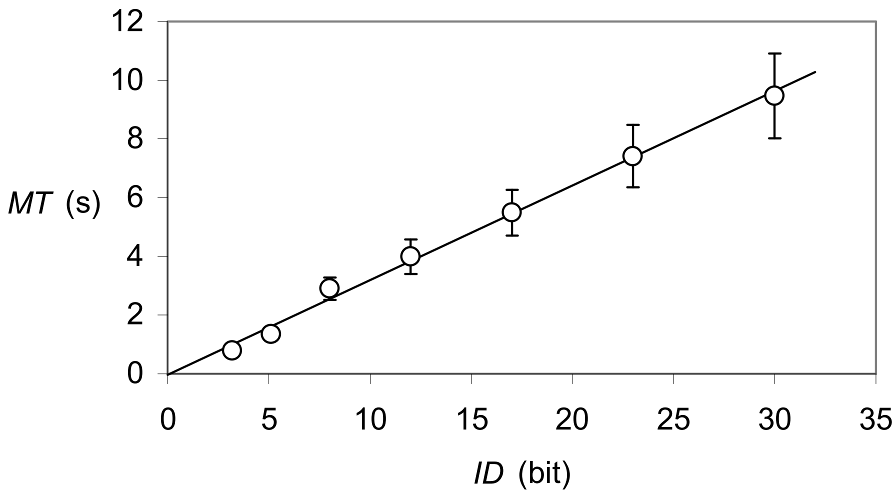


Figure 17. Movement time as a function of the ID, varied up to 30 bits. MTs for the two ID levels where the zoom facility was ignored (3 and 5 bits) are included. Error bars represent 95% confidence intervals based on between-participant standard deviations.

Interestingly, the five data points corresponding to multi-scale pointing (in the 8- to 30-bit range) were found to be aligned with the two leftmost data points, which corresponded to single-scale (i.e., fixed-zoom) pointing. There was no evidence of a difference in the slope of Fitts' law between multi-scale and single-scale pointing (0.30 and 0.29 s/bit, respectively).

Guiard et al.'s study shows that, with multi-scale interfaces, Fitts' law can be investigated far beyond the classic 10-bit barrier without the essence of Fitts' aimed-movement paradigm being altered. Second, with the absolute- and relative-amplitude limits of Fitts' law being removed thanks to the zoom, Guiard et al. were able to show that *MT* varies as a simple logarithmic function of relative amplitude, in strong agreement with Fitts' (1954) prediction.

Perhaps the main potential interest of this exploratory work lies in its opening new avenues for theorizing and experimenting on human aimed movement. The designers of multi-scale interfaces have begun to create radically novel environments for the performance of goal-directed movements and conceived human-environment interaction principles that are without precedents in the real world. In the near future, these technological breakthroughs are likely to elicit conceptual changes in human movement science.

More experiments are obviously needed to accurately model Fitts' law estimated without its traditional limits in the context of zoomable interfaces, but in light of Guiard et al.'s (2000) first findings, modeling of the generalized version of the law as a power relationship, as proposed by Meyer et al. (1990), already seems implausible. Meyer et al. (1990) stated that, by varying the number n of sub-movements involved in an aimed movement, their power model $MT = k_1 + k_2 * (D/W)^{1/n}$ could accommodate both Schmidt et al.'s (1979) linear model (when $n = 1$) and Fitts' (1954) logarithmic model of Fitts' law, arguing that "as n grows larger, this relation approaches a logarithmic function, paralleling Fitts' law" (p. 214). This, however, is a mistake. When n tends to infinity, $1/n$ tends to zero and hence the expression $(D/W)^{1/n}$ tends to unity, yielding $MT =$ a constant. Thus, the model of Meyer et al. (1990) cannot accommodate the logarithmic function that seems to describe Fitts' law in the generalized case with both single-scale and multi-scale aimed movement included.

Second, kinematic[17] analyses of the actual movements of users are needed, in the direction outlined by Furnas and Bederson (1995), to understand pan-zoom coordination.

Recourse to Fitts' paradigm to evaluate target-selection performance in graphic user interfaces has become a norm in HCI (MacKenzie, 1992). Over the past few years, however, studies concerned with multi-scale graphical interfaces in HCI have squarely abandoned Fitts' law, as though inapplicable or irrelevant, and retreated to raw measurements of task completion times. The present work suggests that Fitts' paradigm, as an evaluation tool in HCI, has not said its last word in the face of multi-scale electronic worlds.

Acknowledgments

We gratefully acknowledge the generous financial support of the Programme Cognitique of the French Ministry of National Education and Research (Grant # ACI-COG101 to the first author). The author thanks Michel Beaudouin-Lafon, Shumin Zhai, Reinoud Bootsma, Denis Mottet, Frédéric Bourgeois, Andy Slifkin, and Didier Casalta for helpful discussions.

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[17] More accurately, we are referring to space-scale-time, rather than classic space-time, analyses.

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